# iTaSC concepts and tutorial

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# problem statement

**challenge**

programming general sensor-based robot systems for complex tasks

complex tasks:

- combination of subtasks
- sensor feedback
- large variety of robot systems
- uncertain environments

# problem statement

## current state

- reprogramming for every task
- specialist
- time consuming + expensive

## our goal

development of programming support:

- non-specialists
- less time consuming

# problem statement

## programming support

SYSTEMATIC approach of specification of tasks

## our contribution

framework with:

- systematic approach and
- estimation support for uncertain environments

# aim of presentation

## aim of presentation

- to show, by means of an example application, how framework for ‘Constraint-based task specification and Estimation for Sensor-Based Robot Systems in the Presence of Geometric Uncertainty’ works and what its advantages are
- explain generic control and estimation scheme
- show application to other example tasks

# laser tracing task

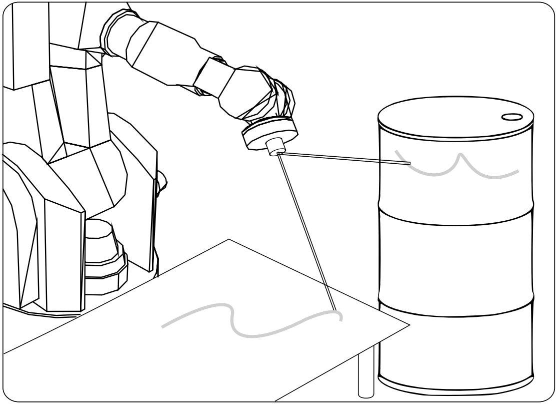

Figure: simultaneous laser tracing on a plane and a barrel

# overview

# general principle

- robot task: accomplishing relative motion and/or controlled dynamic interaction between objects
- specify task by imposing constraints
  $\Rightarrow$ *task function approach* or *constraint-based task programming*

## application independent versus application dependent

- application independent: control and estimation scheme
- application dependent - but systematic: task modeling procedure

# control and estimation scheme

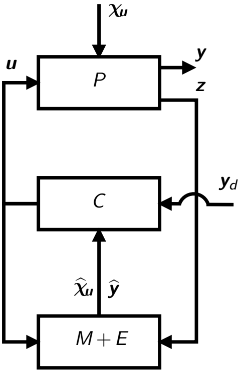


Figure: general control scheme

- plant $P$:
  - robot system ($\boldsymbol{q}$)
  - environment
- controller $C$
- model update and estimation $M+E$

# control and estimation scheme

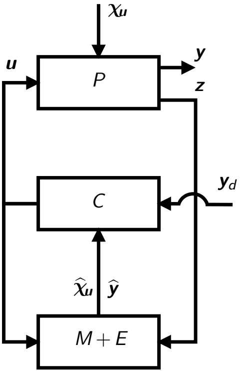


Figure: general control scheme

nomenclature:

- *control input* $\boldsymbol{u}$: desired joint velocities
- *system output* $\boldsymbol{y}$: controlled variables $\Rightarrow$ task specification $=$ imposing constraints $\boldsymbol{y}_d$ on $\boldsymbol{y}$
- *measurements* $\boldsymbol{z}$: observe the plant
- *geometric disturbances*, $\chi_{\boldsymbol{u}}$

# control and estimation scheme

**conclusion**

task independent derivation of
controller block and model update and estimation block
IF
specific *task modeling* procedure is used

# task modeling

- task modeling uses TASK COORDINATES:
- two types of task coordinates:
  - *feature coordinates*, $\chi_f$
  - *uncertainty coordinates*, $\chi_u$
- task coordinates defined in user-defined frames

**goal**

choose frames and task coordinates in a way the task specification becomes intuitive

framework presents procedure to do this

# task modeling procedure

four steps:

1. identify objects and features and assign reference frames
2. choose feature coordinates $\chi_f$
3. choose uncertainty coordinates $\chi_u$
4. specify task

# task modeling procedure

four steps:

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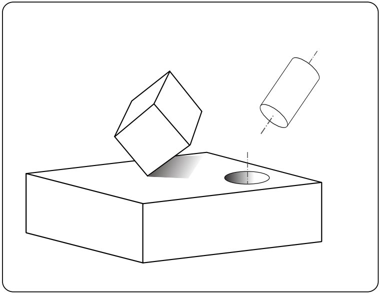
4. specify task

# STEP 1: object and feature frames



a feature is linked to an object

- physical entity
  (vertex, edge, face, surface. . . )
- abstract geometric property
  (symmetry axis, reference frame of a sensor,. . . )

# STEP 1: object and feature frames

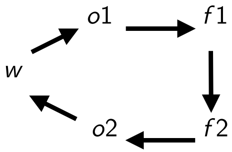



Figure: object and feature frames and feature coordinates

each constraint needs four frames:

- two object frames: $o1$ and $o2$
- two feature frames: $f1$ and $f2$

# STEP 1: object and feature frames

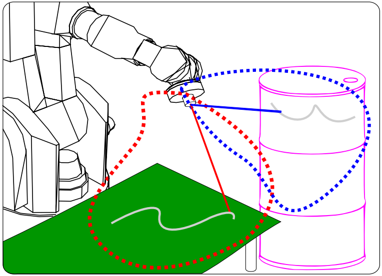
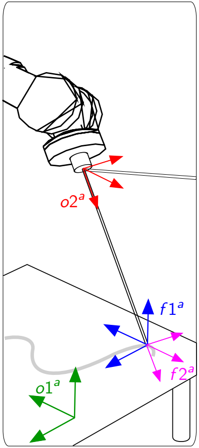


Figure: object and feature frames laser tracing

- natural task description imposes two motion constraints:
  1. trace figure on plane
  2. trace figure on barrel
- ⇒two feature relationships:
  1. feature *a*: for the laser-plane
  2. feature *b*: for the laser-barrel
- the objects are:
  1. laser *a* and laser *b*
  2. the plane
  3. the barrel

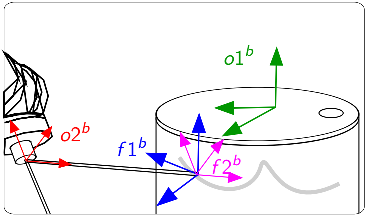
# STEP 1: object and feature frames




**object and feature frames**

- for laser-plane feature:
  - frame $o1^a$ fixed to plane
  - frame $o2^a$ fixed to first laser, $z$-axis along laser beam
  - frame $f1^a$ same orientation as $o1^a$, at intersection of laser with plane
  - frame $f2^a$ same position as $f1^a$ and same orientation as $o2^a$
- for laser-barrel feature:

# STEP 1: object and feature frames




**object and feature frames**

- for laser-plane feature:
- for laser-barrel feature:
  - frame $o1^b$ fixed to barrel, $x$-axis along axis of barrel
  - frame $o2^b$ fixed to second laser, $z$-axis along the laser beam
  - frame $f1^b$ at intersection of laser with barrel, $z$-axis perpendicular to barrel surface, $x$-axis parallel to barrel axis
  - frame $f2^b$ same position as $f1^b$, same orientation as $o2^b$

# task modeling procedure

four steps:

1. identify objects and features and assign reference frames
2. choose feature coordinates $\boldsymbol{\chi_f}$
3. choose uncertainty coordinates $\boldsymbol{\chi_u}$
4. specify task

# STEP 2: feature coordinates

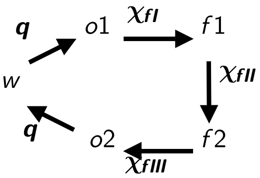
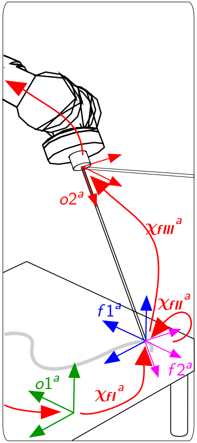



Figure: object and feature frames and feature coordinates

- in general six degrees of freedom between $o1$ and $o2$
- $o1 \rightarrow f1 \rightarrow f2 \rightarrow o2$ = virtual kinematic chain
- for every feature $\boldsymbol{\chi_f}$ can be partitioned

$$\boldsymbol{\chi_f} = \begin{pmatrix} \boldsymbol{\chi_{fI}}^T & \boldsymbol{\chi_{fII}}^T & \boldsymbol{\chi_{fIII}}^T \end{pmatrix}^T \quad (1)$$

# STEP 2: feature coordinates

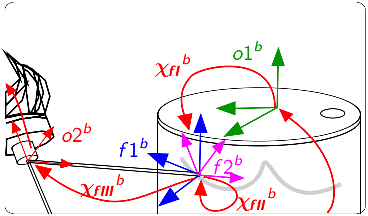



- laser-plane feature:

$$\chi_{fI}{}^a = \begin{pmatrix} x^a & y^a \end{pmatrix}^T \quad (2)$$
$$\chi_{fII}{}^a = \begin{pmatrix} \phi^a & \theta^a & \psi^a \end{pmatrix}^T \quad (3)$$
$$\chi_{fIII}{}^a = \begin{pmatrix} z^a \end{pmatrix} \quad (4)$$

- laser-barrel feature

# STEP 2: feature coordinates




- laser-plane feature
- laser-barrel feature:

$$\chi_{fI}{}^b = \begin{pmatrix} x^b & \alpha^b \end{pmatrix}^T \quad (2)$$

$$\chi_{fII}{}^b = \begin{pmatrix} \phi^b & \theta^b & \psi^b \end{pmatrix}^T \quad (3)$$

$$\chi_{fIII}{}^b = \begin{pmatrix} z^b \end{pmatrix} \quad (4)$$

# task modeling procedure

four steps:

1. identify objects and features and assign reference frames
2. choose feature coordinates $\boldsymbol{\chi_f}$
3. choose uncertainty coordinates $\boldsymbol{\chi_u}$
4. specify task

# STEP 3: uncertainty coordinates

focus on two types of geometric uncertainty:

1. uncertainty pose of object, and
2. uncertainty pose of feature wrt corresponding object

uncertainty *coordinates represent* pose uncertainty of real frame wrt modeled frame:

$$\chi_u = \begin{pmatrix} \chi_{uI}^T & \chi_{uII}^T & \chi_{uIII}^T & \chi_{uIV}^T \end{pmatrix}^T \quad (5)$$

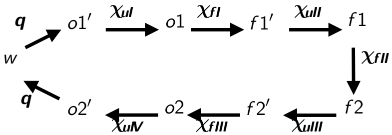
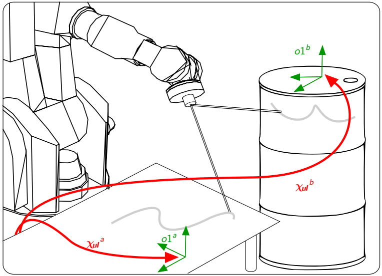



Figure: feature and uncertainty coordinates

# STEP 3: uncertainty coordinates




- unknown position and orientation plane :

$$\chi_{ul}{}^{a} = \begin{pmatrix} h^a & \alpha^a & \beta^a \end{pmatrix}^T$$

- unknown position barrel:

$$\chi_{ul}{}^{b} = \begin{pmatrix} x_u^b & y_u^b \end{pmatrix}^T$$



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# task modeling procedure

four steps:

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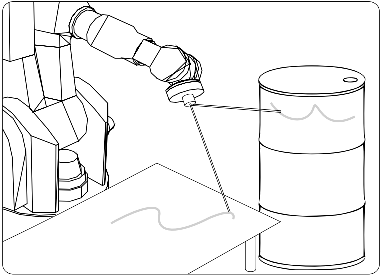
4. specify task

# STEP 4: task specification

**observation**

task is easily specified using task coordinates $\chi_f$ and $\chi_u$



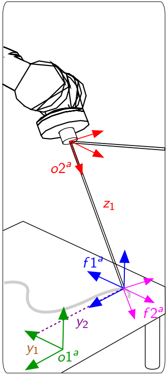
remember: task objective is twofold:

1. trace desired figure on plane
2. trace desired figure on barrel

# STEP 4: task specification

## observation

task is easily specified using task coordinates $\chi_f$ and $\chi_u$




- **output equations:**
  - for the plane:

$$y_1 = x^a \quad \text{and} \quad y_2 = y^a$$

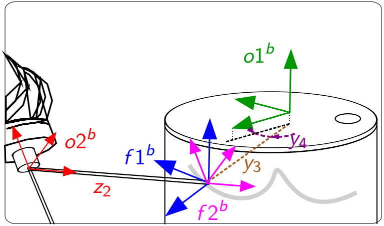
  - for the barrel
- **constraint equations:**
  in this example the desired paths are circles: $y_{id}(t)$, for $i = 1, \ldots, 4$
- **measurement equations:**

$$z_1 = z^a \quad \text{and} \quad z_2 = z^b$$

# STEP 4: task specification

## observation

task is easily specified using task coordinates $\chi_f$ and $\chi_u$




- **output equations:**
  - for the plane
  - for the barrel:

$$y_3 = x^b \quad \text{and} \quad y_4 = \alpha^b$$

- **constraint equations:**
  in this example the desired paths are circles: $y_{id}(t)$, for $i = 1, \ldots, 4$
- **measurement equations:**

$$z_1 = z^a \quad \text{and} \quad z_2 = z^b$$



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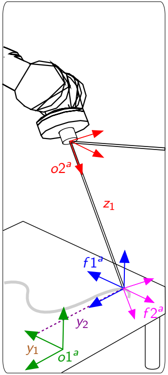
# STEP 4: task specification

## observation

task is easily specified using task coordinates $\chi_f$ and $\chi_u$

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- **measurement equations:**

$$z_1 = z^a \quad \text{and} \quad z_2 = z^b$$



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# STEP 4: task specification

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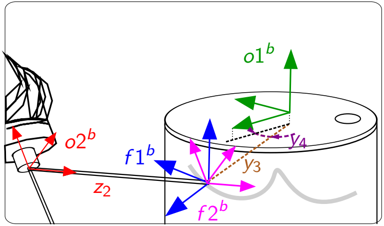


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  in this example the desired paths are circles: $y_{id}(t)$, for $i = 1, \ldots, 4$
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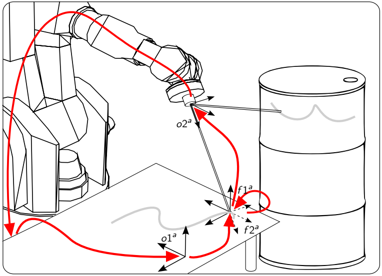
# STEP 4: task specification

## observation

task is easily specified using task coordinates $\chi_f$ and $\chi_u$



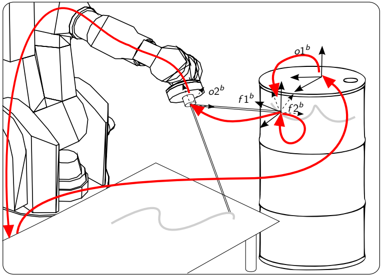

**position loop constraints:**
two position loop constraints, one for each feature relationship

- laser-plane feature *a*
- laser-barrel feature *b*

# STEP 4: task specification

## observation

task is easily specified using task coordinates $\chi_f$ and $\chi_u$




**position loop constraints:**
two position loop constraints, one for each feature relationship

- laser-plane feature *a*
- laser-barrel feature *b*

# task modeling

## conclusion

- application dependent - but systematic modeling procedure provided easy task specification and uncertainty modeling
- application independent controller and model update and estimation block automatically derived

$\Rightarrow$ overall fast and easy task specification

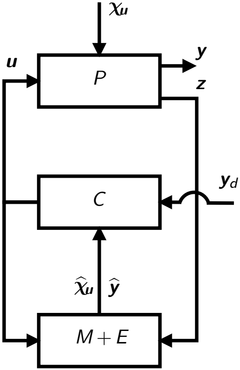



Figure: general control scheme

# overview

# Equations (1)

- *robot system equation:* relates the control input $\boldsymbol{u}$ to the rate of change of the robot system state:

$$\frac{d}{dt}\begin{pmatrix}\boldsymbol{q}\\ \dot{\boldsymbol{q}}\end{pmatrix} = \boldsymbol{s}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{u}) \tag{6}$$

- *output equation:* relates the position based outputs $\boldsymbol{y}$ to the joint and feature coordinates:

$$\boldsymbol{f}\left(\boldsymbol{q}, \chi_{\boldsymbol{f}}\right) = \boldsymbol{y} \tag{7}$$

# Equations (2)

- *measurement equation:* relates the position based measurements $\boldsymbol{z}$ to the joint and feature coordinates:

$$\boldsymbol{h}(\boldsymbol{q}, \chi_{\boldsymbol{f}}) = \boldsymbol{z} \tag{8}$$

- *artificial constraints:* used to specify the task:

$$\boldsymbol{y} = \boldsymbol{y}_d \tag{9}$$

- *natural constraints:* for rigid environments:

$$\boldsymbol{g}(\boldsymbol{q}, \chi_{\boldsymbol{f}}) = \boldsymbol{0} \tag{10}$$

$\rightarrow$ special case of the artificial constraints with $\boldsymbol{y}_d = 0$

# Equations (3)

- dependency relation between $\boldsymbol{q}$ and $\chi_{\boldsymbol{f}}$, perturbed by uncertainty coordinates $\chi_{\boldsymbol{u}}$:

$$\boldsymbol{l}(\boldsymbol{q}, \chi_{\boldsymbol{f}}, \chi_{\boldsymbol{u}}) = \boldsymbol{0} \tag{11}$$

  $\rightarrow$ nonholonomic systems: replace $\boldsymbol{q}$ by operational coordinates $\chi_{\boldsymbol{q}}$
  $\rightarrow$ derived using position closure equations $\Rightarrow$ *loop constraints*

## auxiliary coordinates

the benefit of introducing feature coordinates $\chi_{\boldsymbol{f}}$ is that they can be chosen according to the specific task at hand, such that equations (7)–(10) can much be simplified. A similar freedom of choice exists for the uncertainty coordinates in equation (11)

# control law

## goal

1. provide system input $\boldsymbol{u}$ at each time step

- here: assume a velocity-controlled robot ($\boldsymbol{u} = \dot{\boldsymbol{q}}_d$)
- control law is based on system linearization, resulting in an equation of the form:

$$\boldsymbol{A}\dot{\boldsymbol{q}}_d = \dot{\boldsymbol{y}}_d^{\circ} + \boldsymbol{B}\widehat{\dot{\boldsymbol{\chi}}}_{\boldsymbol{u}} \tag{12}$$

- *weighted pseudo-inverse solving approach* can handle over- and/or underconstrained cases next to constraint weighting: levels of constraints based on nullspace projections
- details in appendix

# model update and estimation

## goal

1. provide estimate for system outputs $\boldsymbol{y}$ used in feedback terms of constraint equations (24)
2. provide estimate for the uncertainty coordinates $\boldsymbol{\chi_u}$ used in control input (26)
3. maintain consistency between joint and feature coordinates $\boldsymbol{q}$ and $\boldsymbol{\chi_f}$ based on the loop constraints

model update and estimation is based on an extended system model:

$$\frac{d}{dt}\begin{pmatrix}\boldsymbol{q}\\ \boldsymbol{\chi_f}\\ \boldsymbol{\chi_u}\\ \dot{\boldsymbol{\chi}}_u\\ \ddot{\boldsymbol{\chi}}_u\end{pmatrix} = \begin{pmatrix}\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} & -\boldsymbol{J_f}^{-1}\boldsymbol{J_u} & \boldsymbol{0}\\ \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0}\\ \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1}\\ \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{q}\\ \boldsymbol{\chi_f}\\ \boldsymbol{\chi_u}\\ \dot{\boldsymbol{\chi}}_u\\ \ddot{\boldsymbol{\chi}}_u\end{pmatrix} + \begin{pmatrix}\boldsymbol{1}\\ -\boldsymbol{J_f}^{-1}\boldsymbol{J_q}\\ \boldsymbol{0}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{pmatrix}\dot{\boldsymbol{q}}_d \qquad (13)$$

# model update and estimation

## prediction-correction procedure

- **prediction**
  1. generate prediction based on extended system model
  2. eliminate inconsistencies between predicted estimates
- **correction**
  1. generate updated estimated based on predicted estimates and information from sensor measurements
  2. eliminate inconsistencies between predicted estimates

# overview

# conclusion (1)

## conclusion

- motion specification and estimation in unified framework
- automatic application independent derivation of control and model update and estimation
- application dependent - but systematic - task modeling

## remark

this presentation focused on the *basic* functionality of the framework
further generalizations include inequality constraints and motion planning

# further reading

## framework journal paper

- Constraint-Based Task Specification and Estimation for Sensor-Based Robot Systems in the Presence of Geometric Uncertainty
- Joris De Schutter, Tinne De Laet, Johan Rutgeerts, Wilm Decré, Ruben Smits, Erwin Aertbeliën, Kasper Claes, and Herman Bruyninckx
- Journal of Robotics Research, May 2007, vol. 26, no. 5, pages 433–455

## extended framework conference paper

- Extending iTaSC to Support Inequality Constraints and Non-Instantaneous Task Specification
- Wilm Decré, Ruben Smits, Herman Bruyninckx, and Joris De Schutter
- Proceedings of the International Conference on Robotics and Automation, 2009, pages 964–971

THANKS FOR YOUR ATTENTION!

# overview

# human-robot co-manipulation

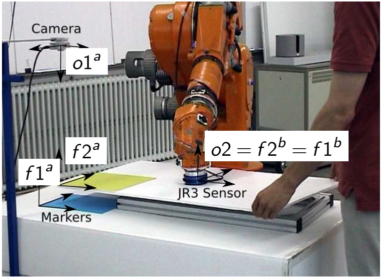



Figure: the experimental setup for the human-robot co-manipulation task

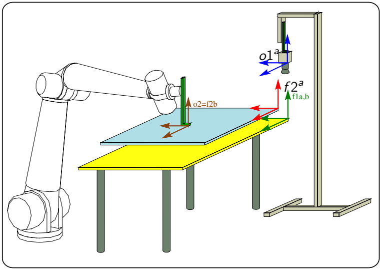



Figure: the object and feature frames for a human-robot co-manipulation task

# object and feature frames

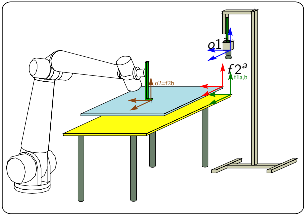



Figure: the object and feature frames for a human-robot co-manipulation task

- natural task description imposes three motion constraints:
  - align one side of the object according to the camera
  - carry the weight and generate downward motion to realize desired contact force
  - follow human intent
- ⇒two feature relationships:
  - feature *a* : visual servoing
  - feature *b*: force control
- the objects are:
  1. the environment (or camera)
  2. the object

# object and feature frames

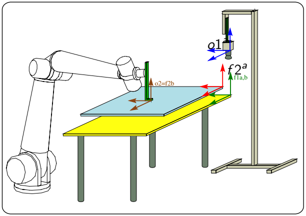



Figure: the object and feature frames for a human-robot co-manipulation task

- frame $o1^a$ fixed to robot environment (camera)
- frame $o2$ at center of object
- $o1b$ fixed to $o2$ by a compliance
- frame $f1^a$ at reference pose on support
- frame $f2^a$ fixed to the object
- no force $\Rightarrow$ frames $f1^b$ and $f2^b$ coincide with $o2$,
  forces $\Rightarrow$ $f1^b$ and $f2^b$ deviate from each other

# feature coordinates

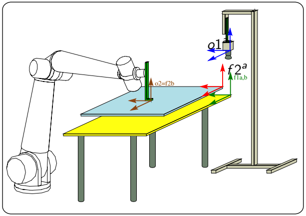



Figure: the object and feature frames for a human-robot co-manipulation task

- for feature $a$:

$$\chi_{fI}{}^{a} = \begin{pmatrix} - \end{pmatrix} \quad (14)$$

$$\chi_{fII}{}^{a} = \begin{pmatrix} x^a & y^a & z^a & \phi^a & \theta^a & \psi^a \end{pmatrix}^T \quad (15)$$

$$\chi_{fIII}{}^{a} = \begin{pmatrix} - \end{pmatrix} \quad (16)$$

- for feature $b$:

$$\chi_{fI}{}^{b} = \begin{pmatrix} - \end{pmatrix} \quad (17)$$

$$\chi_{fII}{}^{b} = \begin{pmatrix} x^b & y^b & z^b & \phi^b & \theta^b & \psi^b \end{pmatrix}^T \quad (18)$$

$$\chi_{fIII}{}^{b} = \begin{pmatrix} - \end{pmatrix} \quad (19)$$

# task specification

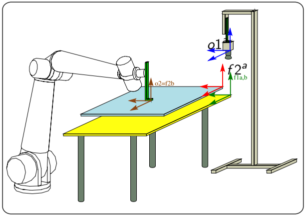



Figure: the object and feature frames for a human-robot co-manipulation task

- **output equations:**
  - camera:
  $$y_1 = x^a, \quad y_2 = y^a \tag{14}$$
  - contact force with support:
  $$y_3 = F_z = k_z x^b, \quad y_4 = T_x = k_{\alpha x}\phi^b, \quad y_5 = T_y = k_{\alpha y}\theta^b \tag{15}$$
  - human interaction:
  $$y_6 = F_x = k_x x^b, \quad y_7 = F_y = k_y y^b, \quad y_8 = T_z = k_{\alpha z}\psi^b \tag{16}$$
- **constraint equations:**
- **measurement equations:**

# task specification

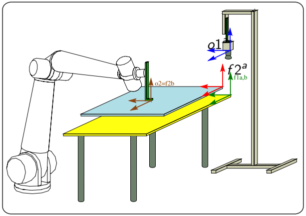



Figure: the object and feature frames for a human-robot co-manipulation task

- **output equations:**
- **constraint equations:**

$$y_{1d} = 0\text{mm}, \quad y_{2d} = 60\text{mm}$$
$$y_{3d} = F_{zd}, \quad y_{4d} = 0, \quad y_{5d} = 0$$
$$y_{6d} = y_{7d} = y_{8d} = 0 \tag{14}$$

- **measurement equations:** in this example all the outputs can be measured:

$$z_i = y_i \quad \text{for } i = 1, \ldots, 8 \tag{15}$$

# results

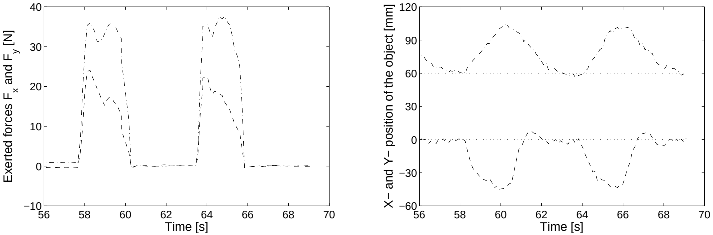



Figure: the left plot shows the forces $F_x$ and $F_y$, exerted by the operator during the co-manipulation task. the right plot shows the alignment errors $x^a$ and $y^a$ as measured by the camera.

# mobile robot

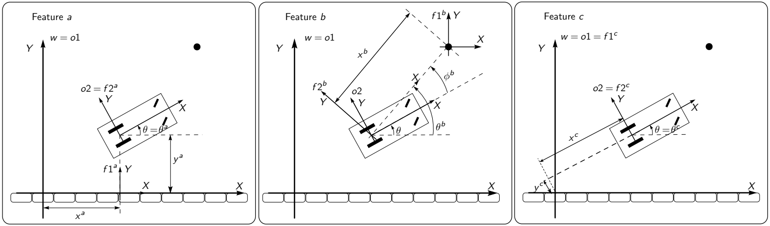



Figure: left for feature $a$, ultrasonic sensor; middle for feature $b$, range finder; right for feature $c$, robot trajectory

# object and feature frames

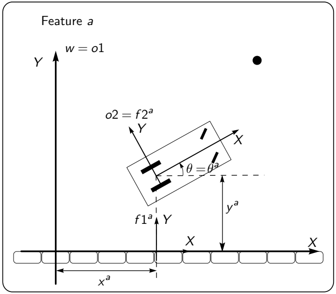



Figure: feature *a*

- task description: move robot along a trajectory with respect to the world while measuring distance to a wall with ultrasonic sensor and measuring the distance and angle to a beacon
- ⇒three feature relationships:
  1. feature *a*: ultrasonic sensor
  2. feature *b*: range finder
  3. feature *c*: motion specification
- the objects are:
  1. mobile robot
  2. environment (wall, beacon)

# object and feature frames

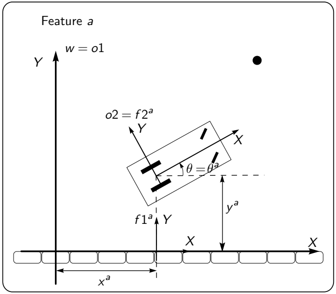



Figure: feature $a$

- frame $o1$, fixed to wall, its $x$-axis along the wall
- frame $o2$, fixed to mobile robot
- for feature $a$ (ultrasonic sensor):
  - frame $f1^a$, same orientation as $o1$ and able to move in $x$ direction of $o1$
  - frame $f2^a$, fixed to frame $o2$

# object and feature frames

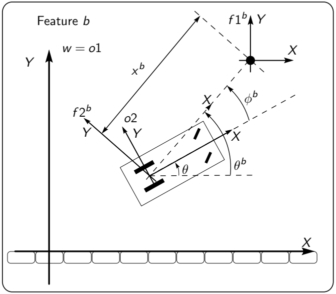



Figure: feature $b$

- frame $o1$, fixed to wall, its $x$-axis along the wall
- frame $o2$, fixed to mobile robot
- for feature $b$ (range finder):
  - frame $f1^b$, at the beacon location, fixed to frame $o1$
  - frame $f2^b$, $x$-axis is beam of range finder hitting the beacon

# object and feature frames

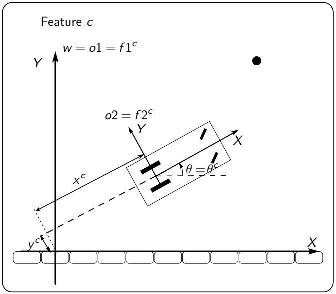



Figure: feature $c$

- frame $o1$, fixed to wall, its $x$-axis along the wall
- frame $o2$, fixed to mobile robot
- for feature $c$ (path tracking):
  - frame $f1^c$, coinciding with $o1$
  - frame $f2^c$, coinciding with $o2$

# feature coordinates

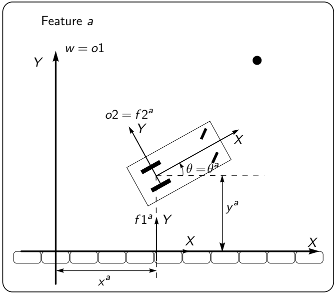



Figure: feature $a$

for each of the three features a minimal set of position coordinates exists representing the 3DOF between $o1$ and $o2$:

- for feature $a$ (ultrasonic sensor):

$$\chi_{fI}{}^a = \begin{pmatrix} x^a \end{pmatrix} \quad (16)$$

$$\chi_{fII}{}^a = \begin{pmatrix} y^a & \theta^a \end{pmatrix}^T \quad (17)$$

$$\chi_{fIII}{}^a = \begin{pmatrix} - \end{pmatrix} \quad (18)$$

# feature coordinates

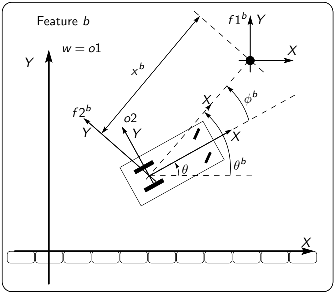



Figure: feature $b$

for each of the three features a minimal set of position coordinates exists representing the 3DOF between $o1$ and $o2$:

- for feature $b$ (range finder):

$$\chi_{fI}{}^{b} = \begin{pmatrix} - \end{pmatrix} \quad (16)$$

$$\chi_{fII}{}^{b} = \begin{pmatrix} x^b & \theta^b \end{pmatrix}^T \quad (17)$$

$$\chi_{fIII}{}^{b} = \begin{pmatrix} \phi^b \end{pmatrix} \quad (18)$$

# feature coordinates

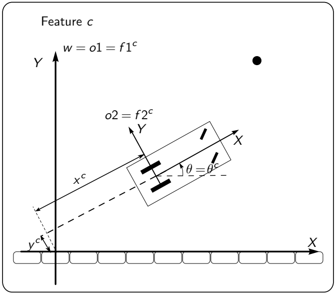



Figure: feature $c$

for each of the three features a minimal set of position coordinates exists representing the 3DOF between $o1$ and $o2$:

- for feature $c$ (path tracking):

$$\chi_{fI}{}^{c} = \left( \; - \; \right) \quad (16)$$

$$\chi_{fII}{}^{c} = \left( \; x^c \quad y^c \quad \theta^c \; \right)^T \quad (17)$$

$$\chi_{fIII}{}^{c} = \left( \; - \; \right) \quad (18)$$

# operational space robot coordinates

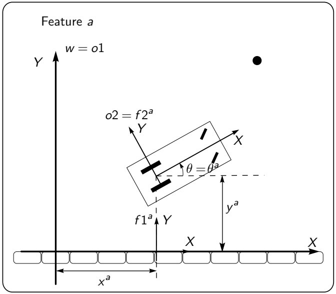



Figure: feature $a$

Nonholonomic robot:

- position loop constraints cannot be written in terms of $\boldsymbol{q}$
- $\Rightarrow$ define operational space robot coordinates $\boldsymbol{\chi_q}$
- natural choice: $\boldsymbol{\chi_q} = \boldsymbol{\chi_f}^c$
- dependency relation between $\dot{\boldsymbol{\chi}}_{\boldsymbol{q}}$ and $\dot{\boldsymbol{q}}$ is very simple: (*nonholonomic constraint*)

$$\dot{\boldsymbol{\chi}}_{\boldsymbol{q}} = \begin{pmatrix} \dot{x}^c \\ \dot{y}^c \\ \dot{\theta}^c \end{pmatrix} = \boldsymbol{J_r}\dot{\boldsymbol{q}} \qquad (16)$$

# uncertainty coordinates

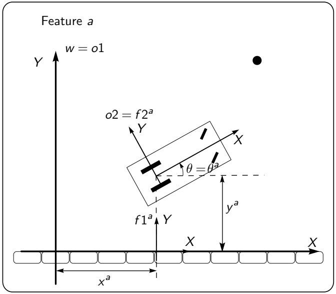



Figure: feature *a*

Nonholonomic robot:

- dependency relation between $\dot{\chi}_{\boldsymbol{q}}$ and $\dot{\boldsymbol{q}}$ is very simple: (*nonholonomic constraint*)

$$\dot{\chi}_{\boldsymbol{q}} = \begin{pmatrix} \dot{x}^c \\ \dot{y}^c \\ \dot{\theta}^c \end{pmatrix} = \boldsymbol{J_r}\dot{\boldsymbol{q}} \qquad (16)$$

- replace $\boldsymbol{q}$ in (7) and (11) by $\chi_{\boldsymbol{q}}$results in:

$$\boldsymbol{C_q} = \frac{\partial \boldsymbol{f}}{\partial \chi_{\boldsymbol{q}}}\boldsymbol{J_r} \quad \boldsymbol{J_q} = \frac{\partial \boldsymbol{I}}{\partial \chi_{\boldsymbol{q}}}\boldsymbol{J_r} \qquad (17)$$

# uncertainty coordinates

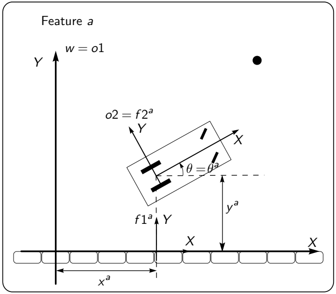



Figure: feature $a$

- the nonholonomic constraint which may be disturbed by wheel slip:

$$\dot{\chi}_{\boldsymbol{q}} = \boldsymbol{J_r}\left(\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}_{slip}\right) \qquad (16)$$

- $\dot{\boldsymbol{q}}_{slip} = s\dot{\boldsymbol{q}}$, with $s$ the estimated slip rate
- $\Rightarrow \chi_{\boldsymbol{uN}} = \boldsymbol{q}_{slip}$ and from (20): $\boldsymbol{J_u} = \boldsymbol{J_q}$

# task specification

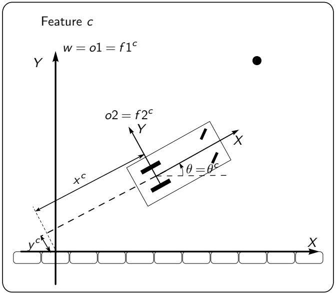



Figure: feature $c$

- **output equations**

$$y_1 = x^c, \quad y_2 = y^c, \quad y_3 = \theta^c.(16)$$

- **constraint equations:**
  from the desired path in terms of $x^a$, $y^a$ and $\theta^a$, the desired values $y_{1d}(t)$, $y_{2d}(t)$ and $y_{3d}(t)$ can be obtained

- **measurement equations:**

$$z_1 = y^a, \quad z_2 = x^b, \quad z_3 = \theta^b \,(17)$$

# feedback control

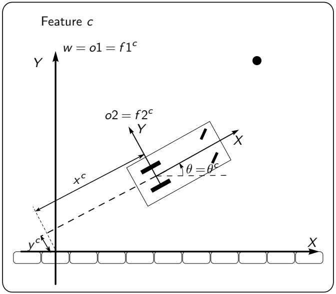



Figure: feature $c$

the path controller is implemented in operation space, by applying constraints (24) with

$$\boldsymbol{K_p} = \begin{pmatrix} k_p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{{k_p}^2}{2sign(\dot{x}_c)} & k_p \end{pmatrix}, \qquad (16)$$

and $k_p$ a feedback constant

# results

without slip:

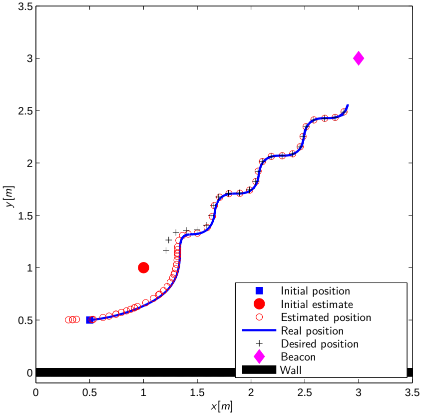
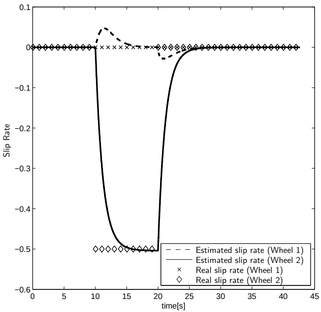


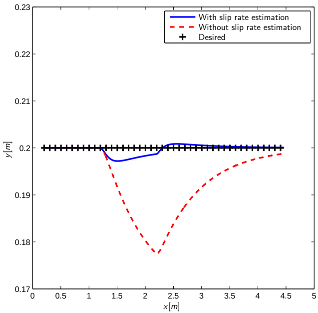

Figure: localization and path tracking control of a mobile robot

# results

with slip:




(a) estimation of the slip rate on both wheels




(b) trajectory of mobile robot

Figure: localization and path tracking control of a mobile robot with slip

# multiple robots with simultaneous tasks

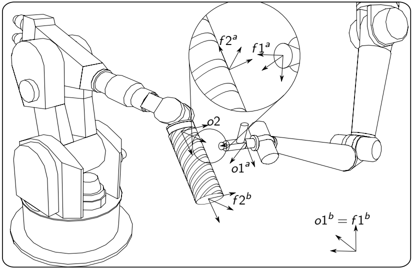



Figure: two robots performing simultaneous pick-and-place and painting operations on a single work piece

# overview

control details

- control law
- closed loop behavior
- invariant constraint weighting

# control law (1)

- differentiate output equation (7) to obtain an output equation at velocity level:

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} + \frac{\partial \boldsymbol{f}}{\partial \chi_{\boldsymbol{f}}}\dot{\chi}_{\boldsymbol{f}} = \dot{\boldsymbol{y}}, \tag{17}$$

written as:

$$\boldsymbol{C_q}\dot{\boldsymbol{q}} + \boldsymbol{C_f}\dot{\chi}_{\boldsymbol{f}} = \dot{\boldsymbol{y}}. \tag{18}$$

- differentiate position loop constraint (11):

$$\frac{\partial \boldsymbol{l}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} + \frac{\partial \boldsymbol{l}}{\partial \chi_{\boldsymbol{f}}}\dot{\chi}_{\boldsymbol{f}} + \frac{\partial \boldsymbol{l}}{\partial \chi_{\boldsymbol{u}}}\dot{\chi}_{\boldsymbol{u}} = \boldsymbol{0} \tag{19}$$

or:

$$\boldsymbol{J_q}\dot{\boldsymbol{q}} + \boldsymbol{J_f}\dot{\chi}_{\boldsymbol{f}} + \boldsymbol{J_u}\dot{\chi}_{\boldsymbol{u}} = \boldsymbol{0} \tag{20}$$

# control law (2)

- $\dot{\chi}_f$ solved from (20):

$$\dot{\chi}_f = -\boldsymbol{J_f}^{-1}\left(\boldsymbol{J_q}\dot{\boldsymbol{q}} + \boldsymbol{J_u}\dot{\chi}_u\right) \tag{21}$$

- substituting (21) into (18) yields the modified output equation:

$$\boldsymbol{A}\dot{\boldsymbol{q}} = \dot{\boldsymbol{y}} + \boldsymbol{B}\dot{\chi}_u \tag{22}$$

  where $\boldsymbol{A} = \boldsymbol{C_q} - \boldsymbol{C_f}\boldsymbol{J_f}^{-1}\boldsymbol{J_q}$ and $\boldsymbol{B} = \boldsymbol{C_f}\boldsymbol{J_f}^{-1}\boldsymbol{J_u}$.

- plant assumed to be ideal velocity controlled system:

$$\dot{\boldsymbol{q}} = \boldsymbol{u} = \dot{\boldsymbol{q}}_d. \tag{23}$$

# control law (3)

- Constraint equation (9) expressed at velocity level and include feedback:

$$\dot{\boldsymbol{y}} = \underbrace{\dot{\boldsymbol{y}}_d + \boldsymbol{K}_p \left( \boldsymbol{y}_d - \boldsymbol{y} \right)}_{\dot{\boldsymbol{y}}_d^{\circ}} \tag{24}$$

- Applying constraint (24) to (22), and substituting system equation (23):

$$\boldsymbol{A} \dot{\boldsymbol{q}}_d = \dot{\boldsymbol{y}}_d^{\circ} + \boldsymbol{B} \widehat{\dot{\boldsymbol{\chi}}}_{\boldsymbol{u}} \tag{25}$$

Solving for the control input $\dot{\boldsymbol{q}}_d$:

$$\dot{\boldsymbol{q}}_d = \boldsymbol{A}_{\boldsymbol{W}}^{\#} \left( \dot{\boldsymbol{y}}_d^{\circ} + \boldsymbol{B} \widehat{\dot{\boldsymbol{\chi}}}_{\boldsymbol{u}} \right) \tag{26}$$

# closed loop behavior

substituting control input (26) in system equation (23) and then in output equation (22), and solving for $\dot{\boldsymbol{y}}$:

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{A}^{\#}_{\boldsymbol{W}}\dot{\boldsymbol{y}}^{\circ}_{d} + \left(\boldsymbol{A}\boldsymbol{A}^{\#}_{\boldsymbol{W}} - \boldsymbol{1}\right)\boldsymbol{B}\dot{\boldsymbol{\chi}}_{\boldsymbol{u}} + \boldsymbol{A}\boldsymbol{A}^{\#}_{\boldsymbol{W}}\boldsymbol{B}\left(\widehat{\dot{\boldsymbol{\chi}}}_{\boldsymbol{u}} - \dot{\boldsymbol{\chi}}_{\boldsymbol{u}}\right) \qquad (27)$$

# invariant constraint weighting

- *pseudo-inverse approach* to handle over- and/or underconstrained cases
- in joint space: mass matrix of robot
- in Cartesian space, $\boldsymbol{W} = diag(w_i^2)$, with:

$$w_i = \alpha \frac{1}{\Delta_{pi} k_{pi}} \quad \text{or} \quad w_i = \alpha \frac{1}{\Delta_{vi}} \tag{28}$$

- next to weighting: levels of constraints based on nullspace projections